

Finite Element Analysis of Unbounded Eddy-Current Problem with Cauer Ladder Network method.

K. Sugahara¹, A. Kameari², H. Ebrahimi², Y. Shindo³ and T. Matsuo⁴

¹Faculty of Science and Engineering, Kindai University, Higashiosaka 577-8502, Japan

²Science Solutions International Laboratory, Inc., Tokyo 153-0065, Japan

³Kawasaki Heavy Industries, Ltd., Akashi 673-8666, Japan

⁴Graduate School of Engineering, Kyoto University, Kyoto 615-8510, Japan

Open boundary techniques are employed to represent the Cauer Ladder Network (CLN) for unbounded Eddy current problems. Recently, a novel and effective model order reduction method, called CLN method, was proposed in which the linear Eddy current fields are represented by the CLN. The CLN method was first formulated for closed domain Eddy current problems. In the paper, it is shown that the CLN method can be extended to the unbounded-domain problems combined with conventional open boundary techniques. As a numerical example, the CLN network parameter extractions of the two parallel round-trip conductors are demonstrated.

Index Terms— Cauer ladder network, eddy current, finite element method, model order reduction, unbounded domain.

I. INTRODUCTION

RECENTLY, a novel and effective model order reduction (MOR) method called Cauer Ladder Network (CLN) method was proposed [1] in which the linear Eddy-current fields are represented by the CLN. The CLN method was first formulated for closed domain Eddy current problems because the orthogonality of the basis functions introduced in the CLN method is only maintained when the surface integral of the Poynting vector on the analysis domain is zero. In practical engineering, however, most of the electromagnetic problems are confined to a finite domain but also to an open boundary problem. A variety of open boundary techniques have been developed for the finite element method to solve unbounded Eddy current problems. Among those techniques, the Kelvin transformation [2, 3] and the Improved Asymptotic Boundary Conditions (IABC) [4-6] emulate open boundaries without the need for any software modification to a standard finite element package and also the surface integrals of the Poynting vector coming out of the analysis region are zero.

In the paper, it is shown with the numerical examples that the CLN method can be extended to the unbounded-domain problems combined with the Kelvin transformation and the IABC.

Figure 1 shows a circuit diagram of the CLN. v and i are the voltage and current of the source. R_{2n} and L_{2n+1} are resistances and inductances of the CLN. e_{2n} and h_{2n+1} are the coefficients of the electric and magnetic basis functions [1]. Figure 2 shows a flowchart of the circuit parameter extraction of the CLN in the A-formulation whereas σ is the conductivity of the conductor; E_0 is the static electric field for unit power supply; J_{2n} is the current density, and A_{2n+1} is the magnetic vector potentials. In the first step, the electric field E_0

generated by unit voltage of the power supply is computed. In the second step, the current density J_0 is computed in the conductors. In the third step, $2n$ -th resistance R_{2n} is computed by taking the integral of the current density over conductors. In the fourth step, the magnetic vector potentials are computed by the FEM analysis with a gauge of $\nabla \cdot \alpha \mathbf{A} = 0$. The gauge is automatically satisfied in the two dimensional analysis.

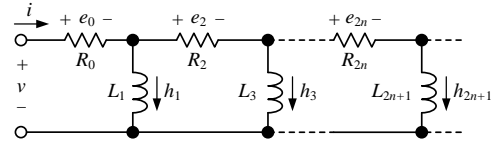


Fig. 1. Circuit diagram of the CLN

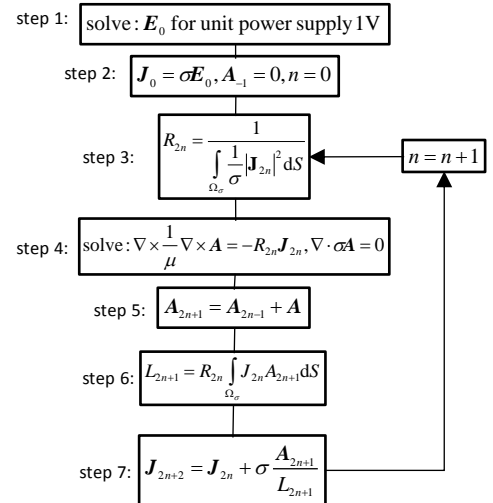


Fig. 2. Flowchart of the circuit parameter extraction of the CLN

In the sixth step, the inductance L_{2n+1} is computed by the taking the square of the magnetic field as (1) in the original formulation,

$$L_{2n+1} = \int_{\Omega} \frac{1}{\mu} |\nabla \times \mathbf{A}_{2n+1}|^2 dS \quad (1)$$

For bounded problems, the surface integral of the Poynting vector on the analysis domain must be zero for the orthogonality of the basis functions introduced in the CLN method to be maintained. For unbounded problems, instead, the integral must be taken over infinite space, however, in the FEM analysis, the analysis region must be truncated with open boundary conditions. To avoid this difficulty, the integral is substituted by (2) whose integrated region is the conducting region only.

$$L_{2n+1} = R_{2n} \int_{\Omega_\sigma} J_{2n} A_{2n+1} dS \quad (2)$$

When the open boundary conditions are perfect like the Kelvin transformation, there are no reflections and the orthogonality is maintained, nonetheless, approximate open boundary conditions always have small reflections which violate the orthogonality of the basis functions. The IABC is the one of the exceptions because the outer most boundary of the IABC is either Dirichlet or Neumann boundary conditions whose surface integral of the Poynting vector is zero. In the seventh step, the current density, J_{2n} is updated by adding the vector potentials and repeated up to the required number of stages of the CLN.

II. NUMERICAL EXAMPLE

Figures 4 show the two parallel round-trip conductors with two-layer and five-layer IABCs. The total thickness of the IABC layer is 0.1 mm and it is divided into slices depending on the number of IABC layers. We employed the general FEM solver “FreeFEM++” [5] to follow the procedure in Fig. 2. The 2nd order elements are employed and the typical mesh size was 0.002 mm generated by the auto mesher bundled in “FreeFEM++”. The permeability of each layer is given in [2]. We have extracted the resistances and inductances of the CLN with various numbers of IABC layers and compared with that with the Kelvin transformation [6] which can be considered as more accurate results.

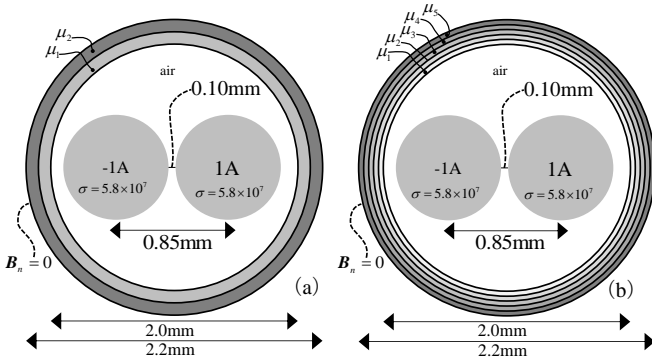


Fig. 3. Two parallel round-trip conductors with (a) two-layer and (b) five-layer IABC.

The obtained results are shown in table 1 and Fig. 5. Figure 5 shows the differences of the circuit parameters between the IABC and the Kelvin transformation. When the number of layers increases, the circuit parameters approach to the ones with the Kelvin transformation. In this example, the differences of the obtained circuit parameters by the four-layer IABC and the Kelvin transformation are below 0.1 %.

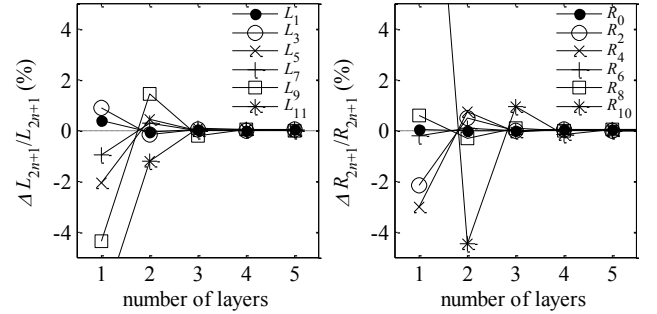


Fig. 4. Differences of the circuit parameters between the IABC and Kelvin transformation.

TABLE 1. CIRCUIT PARAMETERS OF THE CLN

	1-layer IABC	2-layer IABC	3-layer IABC	4-layer IABC	5-layer IABC	Kelvin Trans.
R_0 [Ω]	0.0608	0.0608	0.0608	0.0608	0.0608	0.0608
L_1 [μH]	0.4233	0.4214	0.4218	0.4218	0.4218	0.4218
R_2 [Ω]	0.9098	0.9341	0.9296	0.9298	0.9298	0.9298
L_3 [μH]	0.9850	0.9746	0.9767	0.9765	0.9765	0.9765
R_4 [Ω]	2.594	2.695	2.674	2.675	2.675	2.675
L_5 [μH]	1.824	1.870	1.861	1.862	1.862	1.862
R_6 [Ω]	8.473	8.494	8.489	8.490	8.490	8.490
L_7 [μH]	2.408	2.438	2.431	2.432	2.432	2.432
R_8 [Ω]	24.77	24.55	24.65	24.63	24.63	24.63
L_9 [μH]	4.465	4.736	4.657	4.670	4.668	4.668
R_{10} [Ω]	40.15	31.76	33.54	33.19	33.25	33.24
L_{11} [μH]	10.34	11.00	11.15	11.14	11.14	11.14

REFERENCES

- [1] A. Kameari, H. Ebrahimi, K. Sugahara, T. Matsuo, and Y. Shindo, “Cauer ladder network representation of Eddy-current fields using finite element method for model order reduction,” *Compumag* 2017 ID: 52.
- [2] E. M. Freeman and D. A. Lowther, “A novel mapping technique for open boundary finite element solutions to Poisson’s equation,” *IEEE Trans. Magn.*, vol. 24, no. 6, pp. 2934-2936, 1988.
- [3] E. M. Freeman and D. A. Lowther, “An open boundary technique for axisymmetric and three dimensional magnetic and electric field problems,” *IEEE Trans. Magn.*, vol. 25, no. 5, pp. 4135-4137, 1989.
- [4] D. Meeker, “Improvised open boundary conditions for magnetic finite elements,” *IEEE Trans. Magn.*, vol. 49, no. 10, pp. 5243-5247, 2013.
- [5] K. Sugahara, “Electromagnetic analysis of wireless power transfer system with Improvised Absorbing Boundary Conditions,” *Progress in Electromagnetic Research Symposium*, 2016 Shanghai, 1152-1155.
- [6] K. Sugahara “Improvised Asymptotic Boundary conditions for magneto-static field problems in ellipsoidal and elliptic cylindrical domains,” *IEEE Trans. Magn.*, 2017.
- [7] FreeFEM+ <http://www.freefem.org/>